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Pre-Peak Nonlinearity of Granodiorite in Mohr–Coulomb and Hoek–Brown Models

A. Z. Ter-Martirosyan & G. O. Anzhelo & L. YU. Ermoshina & V. A. Lunin

MSUCE, Department of Soil Mechanics and Geotechnical Engineering, Moscow, Russia
slava.lunin@bk.ru

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Abstract

This paper addresses the use of laboratory-derived deformation parameters of rock specimens for selecting constitutive models in numerical stress–strain analysis. The experimental basis of the study consists of uniaxial compression tests performed on water-saturated cylindrical granodiorite specimens. The tests provided the uniaxial compressive strength, the deformation modulus along the loading branch, the elastic modulus along the unloading branch, the transverse deformation coefficient, and Poisson's ratio.

Analysis of the stress–strain curves shows that the pre-peak behaviour of granodiorite cannot be reduced to a linear elastic response governed by a single constant deformation modulus. A pronounced nonlinear segment is observed at the initial loading stage, which may be associated with the closure of microcracks, pores, and weakened contacts within the rock structure. At the same time, the deformation modulus determined within the standard stress range mainly characterizes the working part of the curve and does not represent the initial deformation stage.

It is shown that the Mohr–Coulomb model and the standard Hoek–Brown model may be used for engineering assessment of limit states and for parametric analysis; however, their application is limited when the objective is to reproduce the complete pre-peak uniaxial compression curve. As a promising direction, the use of models with stiffness evolution is proposed, including modified Hoek–Brown with softening formulations supplemented by a pre-peak deformation modulus evolution law.

Keywords

Granodiorite, Uniaxial compression, Deformation modulus, Mohr–Coulomb, Hoek–Brown.

1 Introduction

Numerical modelling of rock materials requires the selection of a constitutive model capable of describing not only the ultimate strength, but also the deformation behaviour of the material prior to failure. In continuum models of soils and rocks, the stress–strain state is considered as a result of changes in internal variables and material parameters, rather than solely as a function of ultimate strength (Alonso et al. 1990). Therefore, when laboratory-derived parameters are used, it is essential to consider the specific segment of the stress–strain curve from which they were obtained.

For rock materials, one of the most widely used strength relationships is the Hoek–Brown criterion, developed for estimating the strength of intact rocks and jointed rock masses (Hoek and Brown 1980; Hoek and Brown 1997). Its subsequent development through the parameters m_i, GSI, and the disturbance factor D has enabled its broad application in engineering calculations and numerical modelling (Hoek et al. 2002; Hoek and Brown 2019). However, the nonlinear Hoek–Brown strength envelope alone does not ensure reproduction of the complete laboratory stress–strain curve, since the deformational part of standard models is commonly defined by constant values of deformation modulus and Poisson's ratio.

The Mohr–Coulomb model, commonly used as a first-approximation model, is also limited in describing pre-peak nonlinearity, because it assumes a linear elastic response prior to reaching the failure condition. At the same time, the behaviour of rock materials is governed not only by strength, but also by jointing, dilatancy, scale effect, damage accumulation, residual strength, and stiffness evolution during loading (Barton 1976; Barton 1977; Alejano and Alonso 2005; Diederichs 2007; Brown 2008; Cai et al. 2004; Cai et al. 2007; Walton et al. 2019; Brantut and Petit 2022). In this context, Hoek–Brown with softening models are of particular interest, as they allow the degradation of strength parameters and the residual state of the rock mass to be taken into account (Verbilo et al. 2025).

Uniaxial compression tests performed in accordance with modern approaches to determining the strength and deformation moduli of intact rock specimens (ASTM D7012-23 2023) indicate that the calculated deformation modulus often characterizes only a prescribed working stress range. The aim of this study is to assess the applicability of the Mohr–Coulomb and Hoek–Brown models for describing the pre-peak behaviour of water-saturated granodiorite based on laboratory uniaxial compression curves obtained from cylindrical specimens. Particular attention is paid to the relationship between deformation parameters, the shape of the stress–strain curve, and the limitations of models that employ constant elastic stiffness.

2 Experimental Database and Methodology of Analysis

The experimental database includes laboratory tests on water-saturated granodiorite specimens prepared from core samples. Cylindrical specimens of standard geometry were used, and the analysis focused on uniaxial compression curves, which characterize both ultimate strength and stiffness evolution during loading.

The following parameters were determined: uniaxial compressive strength in the water-saturated state $R_{c,ws}$, deformation modulus along the loading branch E_d, elastic modulus along the unloading branch E_y, transverse deformation coefficient ν, and Poisson's ratio μ. The deformation modulus was calculated within the relative stress range $0.05R_c$–$0.5R_c$, which corresponds to the working part of the stress–strain curve but does not include the initial nonlinear deformation stage. This is important when these parameters are used in constitutive models with constant stiffness.

For statistical analysis, a dataset with complete deformation parameters for water-saturated granodiorite was used. For detailed interpretation of curve shape, five representative uniaxial compression curves were selected under comparable conditions in terms of rock type, moisture state, and specimen geometry. In this study, the stress–strain curve is therefore considered not only as a source of input parameters, but also as an independent criterion for assessing the physical suitability of a constitutive model.

3 Correlation Analysis and Deformation Parameters

Correlation analysis was performed to evaluate the relationships between the physical, strength, and deformation characteristics of water-saturated granodiorite. The analysis included complete records with determined deformation parameters. The uniaxial compressive strength in the water-saturated state ranged from 12.54 to 104.24 MPa, the deformation modulus along the loading branch ranged from 3.14 to 38.47 GPa, and the elastic modulus along the unloading branch ranged from 5.82 to 44.11 GPa. This scatter indicates pronounced structural heterogeneity of the granodiorite, even within the same lithological variety.

Table 1. Main correlations between physical and mechanical parameters.

Compared parameters	**Correlation coefficient r**
E_d - E_y	0.959
ν - μ	0.850
τc - $R_{c,ws}$	0.930
$R_{c,w}$s - $R_{c,ad}$	0.618
Depth - $R_{c,ws}$	0.153
ρ - $R_{c,ws}$	-0.121
$R_{c,ws}$ - Ed	0.115

The strongest correlation was observed between the deformation modulus along the loading branch E_d and the elastic modulus along the unloading branch E_y(r=0.959). This confirms a stable relationship between stiffness parameters determined at different stages of testing. However, these parameters are not physically identical: the loading branch includes the effects of microdefect closure and the accumulation of irreversible deformation, whereas the unloading branch mainly reflects the elastic component of the specimen response.

The relationship between uniaxial compressive strength and deformation modulus was weak (r=0.115). This indicates that specimen strength cannot be regarded as a reliable independent predictor of deformation stiffness. Similarly, the weak correlations between density, sampling depth, and strength parameters indicate the limited predictive value of integral physical characteristics.

Thus, the results of the correlation analysis show that selecting a constitutive model based only on individual values of strength, density, or deformation modulus is insufficient. It is necessary to consider the segment of the stress–strain curve from which the deformation parameters were determined. Since Ed is calculated within the range 0.05–0.5Rc, it mainly characterizes the working part of loading and does not describe the initial nonlinear segment.

4 Interpretation of the Curves and Limitations of the Mohr–Coulomb and Hoek–Brown Models

To compare specimens with different strength levels, the normalized stress was used:

$$\eta = \frac{\sigma}{R_c} \tag{1}$$

Where η Relative stress level
σ Current axial stress
R_c Uniaxial compressive strength.

Analysis of the normalized curves shows that the pre-peak behaviour of granodiorite is not strictly linear. At the initial loading stage, a segment with reduced tangent stiffness is observed, after which the curve transitions to a more stable working range. This segment may be associated with the closure of microcracks, pores, and weakened contacts between mineral grains. At this stage, the applied load contributes not only to elastic deformation of the mineral skeleton, but also to rearrangement of the internal structure of the specimen. The normalized curves are shown in Fig. 1.

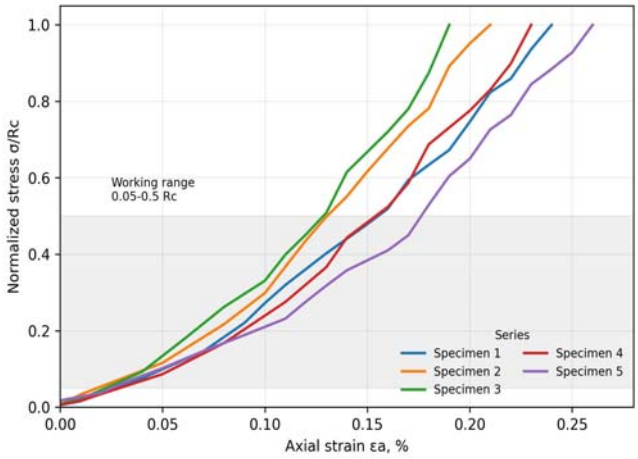


Fig. 1 Normalized uniaxial compression curves of water-saturated granodiorite. The shaded area indicates the initial deformation stage; the range $0.05R_c$–$0.5R_c$ corresponds to the interval used for determining the deformation modulus.

It is important that the deformation modulus E_d, used as a calculation parameter, is not determined over the entire curve, but within the range 0.05–$0.5R_c$. Therefore, it mainly characterizes the working part of loading and does not describe the initial nonlinear segment. If this modulus is directly used in a constitutive model as a constant stiffness parameter, the model will overestimate the stiffness of the specimen at the early loading stage. Conversely, if the modulus is fitted to the initial segment, the calculated stiffness will be underestimated in the main working range.

The Mohr–Coulomb model may be considered a first-approximation model for engineering assessment of rock material behavior. Its advantage lies in the limited number of input parameters and the simplicity of implementation in numerical codes. However, in its classical formulation, the model describes the deformation behavior prior to reaching the limit state using a linear elastic law with constant values of E and ν. For the granodiorite specimens considered in this study, such a formulation cannot reproduce the initial nonlinear segment of the uniaxial compression curve.

The Hoek–Brown model is more consistent with the nature of rock materials in terms of strength description. Its nonlinear strength envelope makes it possible to account for changes in rock resistance at different levels of principal stresses, as well as for the transition from intact rock to rock mass through the parameters m_i, GSI, and D. The generalized version of the Hoek–Brown criterion was specifically intended to facilitate its application in numerical modelling and engineering software; however, the strength envelope itself does not solve the problem of describing the complete laboratory deformation curve. In the standard formulation, the HB model describes strength nonlinearity, but does not introduce an independent law for the evolution of the deformation modulus at the early loading stage.

To account for the initial pre-peak nonlinearity, it is reasonable to introduce a microdefect closure variable χ, varying from 0 to 1 within the initial and working loading ranges:

$$\chi(\eta) = \frac{1 - \exp(-\alpha\eta)}{1 - \exp(-\alpha\eta_w)}, \qquad 0 \le \eta \le \eta_w \tag{2}$$

\Where η_w Upper boundary of the range used to determine the working modulus, η_w=0.5
α Parameter controlling the rate of microdefect closure.
The current deformation modulus can then be expressed as:

$$E(\eta) = E_0 + (E_w - E_0)\chi(\eta) \tag{3}$$

Where E_0 Initial modulus associated with the microdefect closure stage
E_w Working modulus determined in the range 0.05–$0.5R_c$.

The physical meaning of Eq. (3) is that the specimen stiffness is not defined by a single constant value, but gradually increases during the transition from initial structural compaction to the working quasi-elastic segment.

The relationship between stress and axial strain can be written as:

$$\varepsilon_a(\eta) = R_c \int_0^{\eta} \frac{d\xi}{E(\xi)} \tag{4}$$

Eq. (4) makes it possible not only to define a variable modulus, but also to compare the calculated curve with the experimental uniaxial compression curve. In this case, the parameters E_0, E_w, and α can be determined by inverse fitting using the initial and working parts of the laboratory curve. This approach makes the proposed relationship not only a qualitative explanation, but also a potential tool for model calibration.

For the standard Hoek–Brown model, the strength component can be retained in its original form:

$$\sigma_1 = \sigma_3 + \sigma_{ci}(m_b \frac{\sigma_3}{\sigma_{ci}} + s)^a \tag{5}$$

Where σ_1 and σ_3 Principal stresses
σ_{ci} Uniaxial compressive strength of intact rock
m_b, s, a The Hoek–Brown parameters.

Within the proposed approach, Eq. (5) is not replaced, but supplemented by the stiffness evolution law given in Eqs. (2)–(4). This makes it possible to separate two tasks: describing the strength envelope and describing the pre-peak evolution of the deformation modulus.

For Hoek–Brown with softening models, this extension can be formulated using two internal variables: χ, which represents pre-peak microdefect closure, and κ, which represents damage accumulation and post-peak softening. In this case, the strength parameters can be linked to the evolution of GSI:

$$GSI(\kappa) = GSI_r + \big(GSI_p - GSI_r\big)\exp\,(-\beta\kappa) \tag{6}$$

Where GSI_p Initial value of the Geological Strength Index,
GSI_r Residual value
β Degradation rate parameter.

After defining GSI(κ), the parameters m_b, s, and a can be recalculated using the standard Hoek–Brown relationships. This approach is consistent with the general logic of GSI-based rock mass strength assessment, where GSI is used to transition from intact rock to rock mass while accounting for the structure and condition of jointing.

Thus, the proposed extension can be expressed as:

$$E = E(\chi, \kappa), \qquad m_b = m_b(\kappa), \qquad s = s(\kappa), \qquad a = a(\kappa) \tag{7}$$

In this formulation, χ describes the pre-peak stiffness increase associated with microdefect closure, whereas κ represents post-peak degradation caused by damage accumulation. Thus, the proposed scheme separates two physical processes: initial structural compaction and subsequent failure. It should be regarded as a conceptual extension and calibration framework for HB/HBS models rather than a fully verified constitutive model, since the available tests mainly characterize the pre-peak region and do not provide sufficient data for independent calibration of post-peak softening.

5 Conclusions

Laboratory tests on water-saturated cylindrical granodiorite specimens showed that the pre-peak response under uniaxial compression is distinctly nonlinear. The initial loading stage is characterized by reduced tangent stiffness, which may be associated with the closure of microcracks, pores, and weakened contacts within the specimen structure.

The deformation modulus determined along the loading branch within the range 0.05–0.5R_c mainly characterizes the working part of the stress–strain curve and does not capture the initial nonlinear segment. Therefore, using this modulus as a constant stiffness parameter leads to a simplified representation of the deformation behavior of the rock material.

Correlation analysis revealed a strong relationship between the loading deformation modulus and the unloading elastic modulus. However, strength, density, and sampling depth showed weak predictive ability with respect to deformation stiffness, confirming that the shape of the stress–strain curve must be considered in addition to individual parameters.

The Mohr–Coulomb model may be used for preliminary engineering calculations and limit-state assessment, while the Hoek–Brown model provides a more appropriate description of rock strength. Nevertheless, both models are limited in reproducing the complete pre-peak uniaxial compression curve when constant elastic stiffness is assumed.

To improve the physical consistency of the constitutive description, it is proposed to distinguish between pre-peak microdefect closure and post-peak damage accumulation. The former can be described by the variable χ, controlling stiffness evolution, while the latter can be represented by the variable κ, controlling the degradation of GSI, m_b, s, and a within the Hoek–Brown with softening framework.

The proposed scheme does not replace the standard Hoek–Brown criterion, but supplements it with a pre-peak stiffness evolution law. This allows laboratory uniaxial compression curves to be used not only for parameter determination, but also as a basis for selecting and further calibrating constitutive models for rock materials.

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