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Reliability-based design of the rock slope support considering the planar failure of multiple blocks

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Abstract

The assessment of rock slope stability is inherently complex due to the discontinuous nature of rock masses, where failure mechanisms are primarily governed by the geometric and mechanical properties of discontinuities. Traditionally, engineers have relied on deterministic methods based on the concept of factor of safety. However, these approaches often fail to explicitly quantify the uncertainties associated with rock mass properties and geometry. This paper investigates the stability of a multiple-block planar sliding using an analytical system-component formulation. By reducing the potential failure modes of an N-block system to N primary failure modes, the study compares the traditional deterministic design with a reliability-based design approach aligned with the Eurocodes.

The research specifically addresses the influence of geometric uncertainties, considering the dip angle of the sliding surface as a random variable. A comparative analysis was conducted for a slope with six blocks, evaluating the support force required to achieve target reliability indices versus the typical values of the factor of safety. Results indicate that traditional deterministic approaches often overestimate the required support force, serving as an upper bound. Conversely, the equivalent factors of safety derived from the reliability-based design approach vary with the mean dip angle but on average align with the partial factors for the shear strength suggested in Eurocode 7.

Furthermore, the study demonstrates that while multi-block interactions significantly impact stability at lower dip angles, the system tends to behave as a single monolithic volume as the dip angle increases. These findings suggest the adequacy of semi-probabilistic partial factors for idealized rock slope problems while highlighting the benefits of reliability-based methods for design optimization.

Keywords

Rock slope support, Reliability-based design, Planar sliding, Multiple-block system.

1 Introduction

Stability analysis is a fundamental problem in geotechnical engineering, with significant implications for the safety of natural and excavated slopes. While a wide range of analytical and numerical tools is available, the assessment of rock slope stability remains particularly challenging due to the inherently discontinuous nature of rock masses. Unlike in soils, rock mass behaviour is governed by discontinuities, whose geometric and mechanical properties control the kinematics and mechanisms of failure (Hoek and Bray 1981, Goodman 1989).

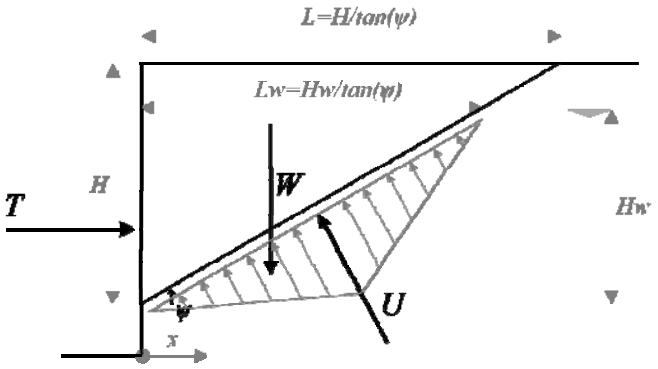
Traditionally, rock slope stability has been evaluated using deterministic approaches based on the factor of safety (*FoS*), in which input parameters are considered as fixed values. Although widely used in practice, this approach does not explicitly account for the uncertainty inherent in rock mass properties. As a result, probabilistic methods have gained interest mostly at the regulation level with a push to include rock mechanics in the scope of Eurocodes (Lamas *et al* 2023). However, the partial factor method, the main approach for limit state verification in the Eurocodes, is complex to be applied to structures so sensitive to geometric properties. In this regard, reliability-based methods have been appointed as potential alternatives given their ability to seek economic optimization under uncertain conditions (Bozorgzadeh and Harrison 2019).

The case of planar sliding of multiple blocks is a sub-problem that given the ability to describe failure mechanisms with analytical formulations, is interesting to be analysed. In this case, the interaction between blocks and the kinetics of the problem can be accounted for in a simpler manner by developing a system-component formulation. This issue was already studied by the authors (Pereira *et al* 2026), and this paper intends to compare the results of this approach with the traditional one and investigate the adequacy of the partial factors in idealized problems, so its application is still possible.

2 Multiple-block planar sliding analysis

Stability analysis regarding planar sliding in rock slopes can easily be carried out considering the limit equilibrium method. Its assumptions are that rock blocks behave as rigid-bodies and the sliding surface is planar and persistent. Fig.1 illustrates the loads acting on a rock block prone to planar sliding due to a set of discontinuities parallel to the slope face outcropping at it. In these conditions, safety is traditionally quantified by the *FoS* given by the ratio between the sliding resistance and the driving actions, i.e.

$$FoS = \frac{(W \cdot \cos\psi - U + T \cdot \sin\psi) \cdot \tan\phi}{W \cdot \sin\psi + T \cdot \cos\psi} \tag{1}$$



Legend:

- H and L are the height and the length of the block.
- H_w and L_w are the height and the length of the portion of the block submersed.
- ψ is the slope dip angle.
- W, U and T are the net forces from the block self-weight, the water pressure and the support, respectively.

Fig. 1 Illustration of the loads acting on a rock block prone to planar sliding.

However, when this potentially unstable volume of rock is divided into several blocks (see Fig. 2), the interaction between those blocks makes the safety quantification more complex, especially in the presence of groundwater, allowing this volume to split apart and only a set of blocks losing stability.

In this case, which may be caused by the presence of another set of discontinuities subvertical and parallel to the slope face, each block presents its own equilibrium state, loading the set of blocks to its left (towards the slope face) if it is unstable or not if it is stable. Such complex loading state is usually analysed using numerical models, with less efficiency. However, in idealized conditions, assuming

uniformity of the shear strength along the sliding surface and rigid-body behaviour of the blocks, physical restraints allow for analytical methodologies for stability analysis to still be carried out.

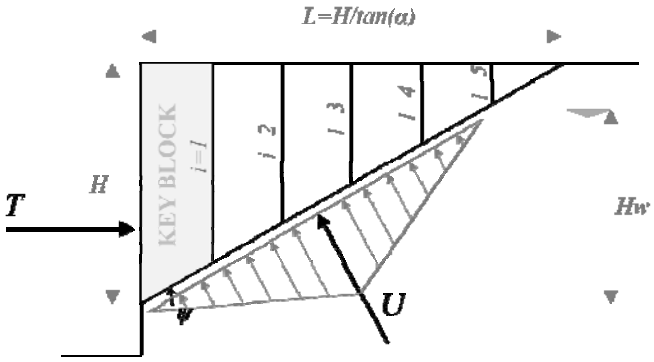


Fig. 2 Illustrative model of a volume of 6 blocks prone to planar sliding.

As mentioned, each block may be itself stable or unstable depending on the loads acting on it and transmitted to it by the block on its right. Considering that the slope instability requires the instability of the key block (the block at the slope free face), there are 2^{N-1} combinations of the stable state of the remaining N-1 blocks, being N the total number of blocks. However, in practice, each block can only become unstable if all the blocks to its left are themselves unstable. Thus, each one of the N-1 discontinuities intersecting the rock volume may divide it into two sets of blocks: a block set A, necessarily unstable, and a block set B, either unstable, loading the former, or stable, reducing the failure modes to (N-1)×2. Furthermore, the situation when block sets A and B slide together is independent of the discontinuity dividing the rock volume. Therefore, the total number of failure modes is reduced to N, corresponding to the instability of *j* blocks at the left, with *j=1,..., N*.In such situation, the representative *FoS* may be the smallest value computed for each failure mode *j*.

In deterministic design of the slope support, the *FoS* is considered a safety indicator, and the intention is to quantify the force needed for the *FoS* to achieve a target value. The solution of this problem can be obtained directly since *FoS* has an analytical expression given in terms of the design variables, in this case the force applied by the support, given by:

$$T = \frac{W \cdot (FoS \cdot \sin\psi - \cos\psi \cdot \tan\phi) + U \cdot \tan\phi}{(FoS \cdot \cos\psi - \sin\psi \cdot \tan\phi)} \tag{2}$$

In reliability-based design, the reliability index β (or the associated probability of failure) is the safety indicator and its target value is often defined in regulations, such as the Eurocodes (EN1990-1 2023). The solution of the problem is obtained through probability theory, expressing the probability of occurrence of failure mode *j* as:

$$P\left[FoS_j < 1\right] \tag{3}$$

Since there are several failure modes involved, due to the existence of several blocks, the rock volume to be analysed is described as a system of blocks and the reliability problem is formulated at the system level. The system fails whenever a failure mode occurs. For each failure mode to occur the stability conditions of sets A and B must be fulfilled. A fully description of this formulation is presented in Pereira *et al* (2026). To calculate the probability of failure of the system a simulation routine was prepared, where each failure mode is checked in turn, incorporating adaptive sampling techniques to speed up the process.

3 Geometric uncertainties

Rock slope stability assessment must account for the inherent uncertainties involved, mostly associated with material properties, resistances, loads and geometric properties. The latter are especially relevant given the sensitivity of the rock mass behaviour to its discontinuous nature (Einstein and Baecher, 1983). In stability assessment of moderately weathered rock masses, failure mechanisms often develop through discontinuities, so its geometric discretization is crucial.

In planar failure of rock slopes, the orientation (dip angle) of the sliding surface is the key factor. The uncertainty on its definition, mostly due to limited site information, must not be ignored. In

probabilistic approaches this uncertainty can be accounted for directly by considering the dip angle as a random variable to which a probabilistic model, such as the von Mises distribution (Fisher, 1953), is assigned. On the other hand, in traditional deterministic approaches a range of plausible values of the dip angle may be established, based on available data, geological context and professional judgement, from which the worst-case scenario is adopted for stability calculations. A hypothetical link between the two may be set by matching the range of plausible values to a confidence interval with a high confidence level.

The selection of the worst-case scenario regarding the dip angle follows a risk-averse behaviour of the rock engineer, who seeks for the most demanding situation with respect to the support needed. For planar sliding, this worst-case scenario is dependent on the external loading. For blocks solely loaded with water pressures, besides their dead weight, it can be showed that the dip angle of the sliding surface that maximizes the support needed (here denoted critical dip angle ψ_c) depends on the targeted *FoS*, being given by:

$$\tan\psi_c = \frac{\tan\phi + \sqrt{\tan^2\phi + FoS^2}}{FoS} \tag{4}$$

where ϕ is the friction angle of the sliding surface. In this loading case, a rock engineer would select the dip angle within the pre-defined range closer to the critical value.

4 Deterministic versus reliability-based design of the support

The design of the slope support follows the definition of a conceptual design model of the ground and the identification of kinematically admissible failure mechanisms. In a context of a rock slope with multiple blocks potentially unstable due to planar sliding, the failure modes were derived in section 2. In this section, a comparison between the traditional deterministic approach based on global factors of safety and the reliability-based approach is intended.

The problem to be tested is described as the stability analysis of a volume of rock, defined by a discontinuity parallel to the slope face and outcropping H=10 meters below the crest, prone to planar sliding (see Fig. 2). Suppose that this volume is intersected by another set of equally spaced vertical discontinuities, parallel to the slope face, forming a system of N=6 blocks. In this situation there are N=6 failure modes associated with the possible instability of larger portions of the rock volume, such as explained before. For simplicity, the water level is fixed at the slope crest and a Mohr-Coulomb envelope with no cohesion is considered for the shear strength of the sliding surface.

To compare the deterministic and reliability-based approaches, some considerations must be made. Firstly, the tangent of the friction angle of the sliding surface is considered as a random variable following a lognormal distribution with mean value of 1 (corresponding to an angle of 45°) and a coefficient of variation of 10%, such as indicated in EN1997-1 (2024). The corresponding characteristic (5^{th} percentile) value of the friction angle, considered in deterministic calculations, is 40.2°. The dip angle is also considered as a random variable following a von Mises distribution with a concentration parameter κ=350 so that the interval ±5° around the mean angle ψ_m corresponds to a 90% confidence interval. In the deterministic approach, for the sake of comparison, this confidence interval is taken as the range of plausible values of the dip angle and the value considered in the calculations is, as mentioned, the one within this interval closer to the critical dip angle. For generalization purposes, the analysis was repeated for mean dip angle ψ_m ranging from 15° to 75°.

For the reliability-based approach, the target reliability indices considered are those indicated in the Eurocodes (EN1990-1 2023), namely 3.3, 3.8 and 4.3 for structures of consequence classes CC1, CC2 and CC3, respectively. With this respect, a direct comparison to the deterministic approach is not possible since there are no settled criteria that establish the target factor of safety. In fact, this factor, which intends to include several sources of uncertainty, is set subjectively by the designer, although the range from 1.3 to 1.5 is often seen as a “rule of thumb” (Wyllie and Mah 2005).The reliability-based design is performed iteratively using the Monte Carlo simulation with adaptive sampling and the deterministic design is computed directly from equation (2) for each failure mode and then assume the larger value of the support as the design value. Fig. 3 shows the results, where the grey-shadowed area corresponds to the solution of the deterministic design (DD), considering the range of *FoS* from 1.3 to

1.5, and the three coloured lines correspond to the solution of the reliability-based design (RBD) for consequence classes CC1, CC2 and CC3.

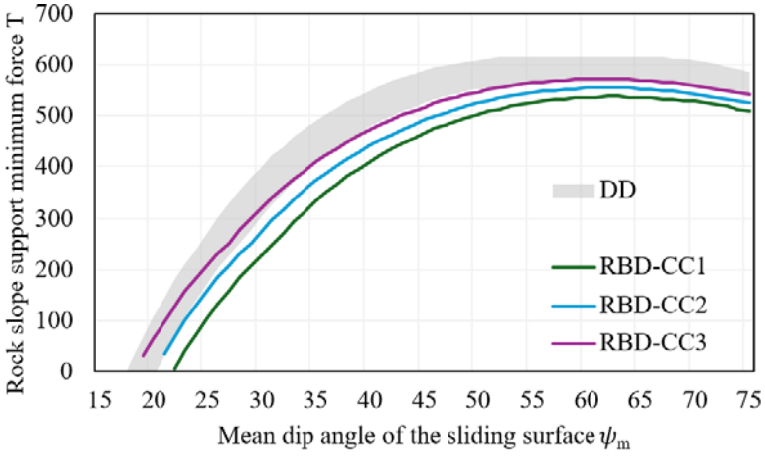


Fig. 3 Rock slope support minimum force obtained following the deterministic and the reliability-based approach.

The range of values of the *FoS* tend to overestimate the minimum force of the rock slope support, indicating that 1.3 to 1.5 are an upper bound for possible values to be adopted as partial factors in the semi-probabilistic approach. In this respect, Fig. 4 shows the equivalent *FoS* that would be obtained by considering the values of the support force computed following the reliability-based design approach. These values were calculated after an inverse analysis, solving Eq. (1) using the support force obtained in the reliability-based design.

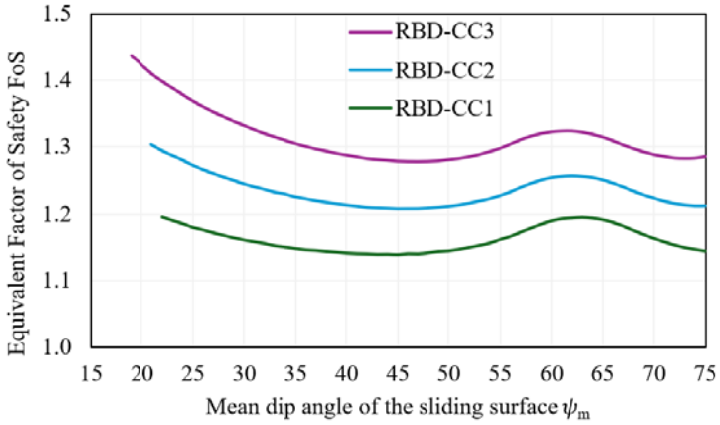


Fig. 4 Factor of safety computed considering the support force obtained in the reliability-based design.

Although the strategy implemented in deterministic calculations to account for the uncertainty of the dip angle can control the variation of the safety level (it inflects for dip angles greater than the critical dip angle ψ_c, roughly between60° and 65°), the equivalent *FoS* still varies with the mean dip angle of the sliding surface. However, the partial factors for the shear strength suggested in Eurocode 7 (EN1997-3 2025), of 1.125, 1.25 and 1.375 for structures of consequence classes CC1, CC2 and CC3, respectively, grossly matches an average value of the equivalent *FoS* determined, suggesting its adequacy. It should however be noted that other situations regarding the loading conditions of the slope, namely external loading and variable water levels, may put this conclusion in question.

Another aspect addressed was the effect of considering a more complex design model, accounting for a set of discontinuities intersecting the volume of rock and generating several blocks. A comparison of the support force determined for the multi-block model with the one obtained in the simpler case that only accounts for the instability of the whole volume can be found in Fig. 5. In fact, differences are only perceived for lower values of the mean dip angle. In fact, as the dip angle increases the possibility for the volume of rock to split apart and some blocks (set A) lose stability while others (set B) remain stable becomes unlikely. The water pressures installed in the vertical joints between the two block sets is less capable of withstanding the weight of block set B, which more easily slides together with block set A.

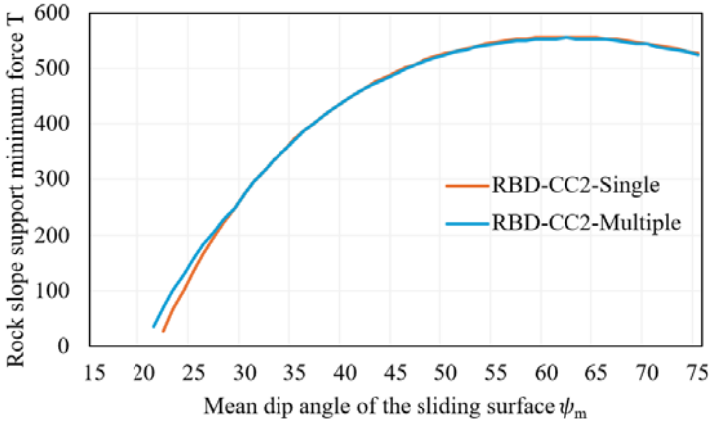


Fig. 5 Rock slope support minimum force for a multi-block and single block models.

5 Conclusions

This study demonstrates that rock slope stability analysis significantly benefits by changing from deterministic to reliability-based design. By modelling the rock mass as a system of multiple interacting blocks, this research captures complex failure modes that traditional single-block models overlook, particularly at lower dip angles. The comparison reveals that the conventional factor of safety range of 1.3 to 1.5 often leads to conservative and consequently uneconomical support designs. Conversely, the partial factors suggested in Eurocode 7 (1.125 to 1.375) were found to be adequate, as they closely match the average equivalent factor of safety derived from probabilistic analyses, validating their adequacy for idealized planar sliding scenarios. Ultimately, incorporating geometric and material uncertainties through probabilistic methods provides an optimized approach to geotechnical safety.

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